Universal Access

The California Common Core State Standards for Mathematics (CA CCSSM) articulate rigorous grade-level expectations. These common standards provide an historic opportunity to improve access to rigorous academic content for all students, including students with special needs. All students should be held to the same high expectations outlined in the mathematics practice and content standards, though some students may require additional time, language support, and appropriate instructional support as they acquire knowledge of mathematics. Effectively educating all students requires diagnosing each student instructionally, adjusting instruction accordingly, and closely monitoring student progress. Regular and active participation in the classroom—not only reading and listening but also discussing, explaining, writing, representing, and presenting—is critical to success in mathematics.

The sections that follow address the instructional needs of many California students in an overarching manner. While suggestions and strategies for mathematics instruction are provided throughout this chapter, they are not intended to—nor could they be expected to—offer teachers and other educators a road map for effectively meeting the instructional needs of each student. Not only do the instructional needs of each student differ from others, the instructional needs of individual students change over time and throughout their mathematics learning progression. Therefore, high quality curriculum, purposeful planning, flexible grouping strategies, differentiation, and progress monitoring are essential components of ensuring universal access.

The first sections in this chapter concentrate on instruction for a broad range of students and include discussions on instructional design, the new language demands of the CA CCSSM, assessment to identify instructional needs, systems of support, and strategies for differentiation. The later sections focus on students with targeted instructional needs: students with disabilities, English learners, Standard English learners, at-risk learners, and advanced learners.

Planning for Universal Access

The ultimate goal of mathematics programs in California is to ensure universal access to high-quality curriculum and instruction so that all students are college and career ready. Through careful planning for modifying their curriculum, instruction, grouping, and assessment techniques, teachers can be well prepared to adapt to the diversity in their classrooms. Universal Access in education is a concept which utilizes strategies for planning for the widest variety of learners from the beginning of the lesson design and not “added on” as an afterthought. Universal Access is not a set of curriculum materials or specific time set aside for additional assistance but rather a schema. For students to benefit from universal access, teachers may need assistance in planning instruction, differentiating curriculum, infusing Specially Designed Academic Instruction in English (SDAIE) techniques, using the California English Language Development Standards (CA ELD standards), and using grouping strategies effectively. Teachers need to utilize many strategies to meet the needs of all of their students

Strategies that may be useful in planning for universal access include:

- Assess and/or screen each student's mathematical skills and understandings at the start of instruction to uncover strengths and weaknesses (pre-test).
- Assess or be aware of the language development level of English learners.
- Utilize formative assessments on an ongoing basis to modify instruction and reevaluate student placement.
- Create a safe environment and encourage students to ask questions.
- Draw on students' previous experiences or cultural relevance as guides.
- Engage in careful planning and organization with the various needs of all learners in mind and in collaboration with specialists (teachers of special education and English learners).
- Engage in backwards and cognitive planning[1] to compensate for skill deficits and to redirect of common misunderstandings.
- Organize lessons in a manner that includes sufficient modeling and guided practice before moving to independent practice.
- Differentiate instruction when necessary, focusing on the mathematics standards and the concepts within the standards.
- Pre-teach routines of classroom grouping and procedures.
- Utilize the *Progressions Documents for the Common Core Math Standards (Progressions)* to understand how mathematical concepts are developed

[1] Backwards planning identifies key areas such as prior knowledge needed, common misunderstandings, organizing information, key vocabulary, and student engagement. Backwards planning is what will be included in a lesson or unit to support intended student learning. Cognitive planning focuses on how instruction will be delivered, anticipates potential student responses and misunderstandings, and provides opportunities to check for understanding and reteaching during the delivery of the lesson. Backwards planning determines what elements will be included; cognitive planning determines how those elements will be delivered.

throughout the grades and to identify strategies to address individual student needs. The *Progressions* documents can be accessed at http://ime.math.arizona.edu/progressions/.

- Explain concepts and procedures using multiple representations that can be displayed through drawings, manipulatives, and/or technology.
- Allow students to demonstrate their understanding and skills in a variety of ways.
- Employ flexible grouping strategies.
- Provide opportunities for students to collaborate and engage in mathematical discourse.
- Include activities that allow students to practice oral discussion of concepts and thinking.
- Emphasize academic and discipline-specific vocabulary.
- Provide students with language models and structures for "speaking mathematics."
- Use sentence frames (communication guides) to support academic vocabulary and language learners.
- Enlist help from other teachers, curriculum specialists, and other specialists (e.g., teachers of special education and English learners).
- Explore technology or other instructional devices.
- Deepen or accelerate student learning.

Additional suggestions to support students who have learning difficulties are provided in "Appendix F. Possible Adaptations for Students with Learning Difficulties in

Mathematics." This list of possible adaptations addresses a range of students, some of whom may have identified instructional needs and some who are struggling unproductively for undiagnosed reasons. If a student has an IEP or 504 Plan, the strategies, accommodations, or modifications in the plan guide the teacher on how to differentiate instruction and additional adaptions should be used only when they are consistent with the IEP or 504 Plan.

Universal Design for Learning

Universal Design for Learning (UDL) is a framework for implementing the concepts of Universal Access by providing equal opportunities to learn for ALL learners. The principles of Universal Design for Learning (UDL) support access to all aspects of learning for all students. Based on the premise that one-size-fits-all curricula create barriers to learning for many students, including the mythical "average" student, UDL helps teachers design curricula to meet the varied instructional needs of all of their students. The goal of UDL curricula is to help students become "expert learners" who are, "…a) strategic, skillful, and goal directed; b) knowledgeable; and c) purposeful and motivated to learn more" (Center on Applied Special Technology 2011, 7). Universal Design for Learning is in alignment with the Standards for Mathematical Practice.

The UDL Guidelines developed by the Center on Applied Special Technology (CAST) are strategies to help teachers make curricula more accessible to all students. The guidelines are based on three primary principles of UDL and are organized under each of the principles as follows. (For more information on UDL, including explanations of the principles and guidelines and the detailed checkpoints for each guideline, go to the National Center on Universal Design for Learning Web page at http://www.udlcenter.org/aboutudl/udlguidelines.)

Principle I: Provide Multiple Means of Representation (the “what” of learning)

Guideline 1: Provide options for perception

Guideline 2: Provide options for language, mathematical expressions, and symbols

Guideline 3: Provide options for comprehension

The first principle, Multiple Means of Representation, allows flexibility so that mathematical concepts can be taught in a variety of ways to address the background knowledge and learning needs of students. For example, presentation of content for a geometry lesson could utilize multiple media that includes written, graphic, audio, and technological formats. Similarly, the presentation of content will include a variety of lesson formats, instructional strategies, and student grouping arrangements (Miller 2009, 493).

Principle II: Provide Multiple Means of Action and Expression (the “how” of learning)

Guideline 4: Provide options for physical action

Guideline 5: Provide options for expression and communication

Guideline 6: Provide options for executive functions

The second principle, Multiple Means of Action and Expression, allows for flexibility in how students demonstrate understanding of the mathematical content. For example, when explaining the subtraction algorithm, fourth-grade students may use concrete materials, draw diagrams, create a graphic organizer, or deliver an oral report or a multimedia presentation (Miller 2009, 493).

Principle III: Provide Multiple Means of Engagement (the “why” of learning)

Guideline 7: Provide options for recruiting interest

Guideline 8: Provide options for sustaining effort and persistence

Guideline 9: Provide options for self-regulation (CAST)

The purpose of the third principle, Multiple Means of Engagement, is to ensure that all students maintain their motivation to participate in the mathematical learning. Alternatives are provided that are based upon student needs and interests, as well as for “(a) the amount of support and challenge provided, (b) novelty and familiarity of activities, and (c) developmental and cultural interests” (Miller 2009, 493). Assignments provide multiple entry points with adjustable challenge levels. For example, sixth-grade students may gather, organize, summarize, and describe distributions for a statistical question at their level of mathematical understanding. In order to develop self-regulation, students reflect upon their mathematical learning through journals, check sheets, learning logs, or portfolios, and are provided encouraging and constructive feedback from the teacher through a variety of formative assessment measures that demonstrate student strengths and areas where growth is still necessary.

While developing curriculum and planning instruction based on UDL principles will require considerable time and effort, all students can benefit from an accessible and inclusive environment that reflects a universal design approach. Teachers and other educators should be provided professional learning on universal design for learning, time for curriculum development and instructional planning, and the necessary resources (e.g., equipment, software, instructional materials) to effectively implement universal design for learning. For example, interactive whiteboards can be a useful tool for providing universally designed instruction and engaging students in learning. The teacher and the students can use the whiteboard to explain a concept or illustrate a procedure. The large images projected on the board that can be seen by most students, including those with visual disabilities (DO-IT 2012).

New Language Demands of the CA CCSSM

As the CA CCSSM are implemented, students will face increased language demands during mathematics instruction. Students will be asked to engage in discussions on mathematics topics, to explain their reasoning, to demonstrate their understanding, and to listen to and critique the reasoning of others. The increased language demands may pose challenges for all students and even greater challenges for English learners. These expectations are made explicit in several of the standards for mathematical practice. **MP.3**, “Construct viable arguments and critique the reasoning of others,” states that students will justify their conclusions, communicate them to others, and respond to the arguments of others. It also states that students at all grades can listen to or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve arguments. **MP.6, “**Attend to precision,” states that students try to communicate precisely with each other, to use clear definitions in discussions with others and in their own reasoning, and that even in the elementary grades students offer carefully formulated explanations to each other. **MP.1**, “Make sense of problems and persevere in solving them,” states that students can explain correspondences between equations, verbal descriptions, tables and graphs.

Standards that call for students to describe, explain, demonstrate and understand provide opportunities for students to engage in speaking and writing about mathematics. These standards appear at all grade levels. For example in grade two, standard **2.OA.9** asks students to explain why addition and subtraction strategies work. In the conceptual category of algebra, standard **A-REI.1** requires students to explain each step in solving a simple equation and to construct a viable argument to justify a solution method.

To support students’ ability to express their understanding of mathematics, teachers need to explicitly teach not only the language of mathematics but also academic language for argumentation (proof, theory, evidence, in conclusion, therefore), sequencing (furthermore, additionally) and relationships (compare, contrast, inverse, opposite) depending on both their English language development level and academic level. Teachers should use the CA ELD standards as a guide to understand the instructional needs of English learners. Pre-teaching vocabulary and key concepts allows students to be actively engaged in learning during the lesson. To help students organize their thinking, teachers may need to scaffold both with graphic organizers and with sentence frames (also called communication guides).

As the CA CCSSM are implemented, students will read and write in mathematics to support their learning. According to Bosse and Faulconer, “Students learn mathematics more effectively and more deeply when reading and writing is directed at learning mathematics” (Bosse 2008, 8). Mathematics text is informational text that requires different skills to read than narrative texts. The pages in a mathematics textbook or journal article can include text, diagrams, tables, and symbols that are not necessarily read from left to right. Students may need specific instruction on how to read and comprehend mathematics texts.

Writing in mathematics also requires different skills than writing in other subjects. Students will need instruction in writing informational/explanatory text that requires facility with the symbols of mathematics and graphic representations in addition to understanding of mathematical content and concepts. Instructional time and effort focused on reading and writing in mathematics benefits students by “requiring them to investigate and consider mathematical concepts and connections…” (Bosse, 2008, page 10), which support the mathematical practices standards. Writing in mathematics needs to be explicitly taught as not all the skills necessary can be automatically transferred from English language arts or English language development. Therefore, students benefit from modeled writing, interactive writing, and guided writing in mathematics.

As teachers and curriculum leaders design instruction to support students’ reading, writing, speaking, and listening in mathematics, the Common Core State Standards for

English Language Arts and Literacy in History/Social Studies, Science, and Technical Subjects (CA CCSS for ELA/Literacy) and the California English Language Development (CA ELD) standards, adopted by the State Board of Education in November 2012 (CDE 2013, http://www.cde.ca.gov/sp/el/er/eldstandards.asp), are essential resources. The standards for reading informational text specify the skills students must attain to be able to comprehend and apply what they read. The writing standards for informational text, in particular Standard 2 in the Writing strand, provide explicit guidance on writing informational/explanatory texts by clearly stating the expectations for students' writing. Engaging in mathematical discourse can be challenging for students who have not had many opportunities to explain their reasoning, formulate questions, or critique the reasoning of others. Standard 1 in the Speaking and Listening strand of the CA CCSS for ELA/Literacy and Part I of the CA ELD standards calls for students to engage in collaborative discussions and sets expectations for a progression in the sophistication of student discourse from kindergarten through grade twelve and from the Emerging level to the Bridging level for English learners. Teachers and curriculum leaders can utilize Standard 1 and Part I as starting points for helping students learn how to participate in mathematical discourse. In grades six through twelve, there are standards for literacy in science and technical subjects that include reading and writing focused on domain-specific content and can provide guidance as students are required to read and write more complex mathematics text.

The CA ELD standards are an important tool for designing instruction to support students' reading, writing, speaking, and listening in mathematics. The CA ELD standards help guide curriculum, instruction, and assessment for English learners who are developing the English language skills necessary to engage successfully with mathematics. California's English learners (ELs) are enrolled in a variety of different school and instructional settings that influence the application of the CA ELD Standards. The CA ELD standards apply to all of these settings and are designed to be used *by all teachers of academic content and of English language development in all these settings*, albeit in ways that are appropriate to the setting and identified student needs. Additionally, the CA ELD Standards are designed and intended to be used *in tandem with* the CA CCSSM to support ELs in mainstream academic content classrooms.

Just as the CA CCSSM should not be treated as a checklist, neither should the CA ELD standards; instead they should be utilized as a tool to equip ELs to better understand mathematics concepts and solve problems. Factors affecting EL students' success in mathematics should also be taken into account. (See the section on course placement, below.) There are a multitude of such factors that fall into at least one of seven characteristic types.

These types of factors affect the success of ELs in mathematics:

- Limited prior and/or background knowledge
    - Some ELs may lack basic mathematics skills and the ability to grasp the new mathematics concepts taught. EL students with limited prior

schooling may not have the basic computation skills required to succeed even in the first year of higher mathematics. Some ELs may have this prior/background knowledge but it is important to avoid misconceptions of students' mathematics skills levels, especially when based upon their cultural background and upbringing.

- Cultural differences
  - Mathematics is often considered to be a universal language where numbers connect people regardless of culture, religion, age, or gender ("ELLs and Mathematics"). However, learning styles vary by country as well as individually. Some ELs may have little or no experience working in cooperative groups or sharing and discussing the solution of a problem.
  - Some symbols' meanings, such as commas and decimal points, and mathematical concepts differ according to culture and country of origin. This occurs frequently, especially when expressing currency values, measurement, temperature, etc., and impedes an EL's understanding of the material being taught. Early on in the school year, teachers should survey their students and learn about their backgrounds to effectively address their needs.
- Linguistics
  - Everyday language is very different from academic language and ELs experience acquisition difficulties when trying to understand and apply these differences. Some of these challenges are understanding

mathematics vocabulary that is difficult to decode and specific to mathematics, associating mathematics symbols with concepts and the language used to express those concepts, grasping the complex and difficult structure of passive voice (frequently used in word problems that are searching for an unknown number), and comprehending strings of words used to create complex phrases with specific meanings (e.g., square root, measure of central tendency).

- Polysemous words
    - Polysemous words have the same spellings and pronunciations but the meanings are different based on context. For example, a “table” is a structure on which one can set food and dishes but it is also something in which one can place data and information. An “operation” is a medical procedure but it is also a mathematical procedure. These meanings are different from each other in context, but the meanings do have some relation to each other. The difference between polysemes and homonyms are subtle. While polysemes have semantically related meanings, homonyms do not.
    - Many words used in mathematics differ from their everyday life meanings. This can be confusing to ELs and may take time to understand. The instruction of specific vocabulary is crucial because vocabulary knowledge correlates with mathematics reading comprehension (“ELLs and Mathematics”).
- Syntactic features of word problems

- o The arrangement of words in a sentence plays a major role in understanding phrases, clauses, or sentences. Faulty syntax is especially detrimental in the reading, understanding, and solving of word problems in mathematics (“ELLs and Mathematics”). Extra support should be given to ELs regarding syntactic features.
- o Some algebraic expressions are troublesome for ELs because they should not be translated word for word. For example: *The number “x” is 5 less than the number “y”.* It is logical to translate word for word when solving this problem and that would be an EL student’s first instinct. This would most likely result in the following translation: $x=5-y$. However, the correct answer would be $x=y-5$.

• Semantic features

- o Many ELs find semantic features challenging, such as:

FEATURE	EXAMPLE
Synonyms	add, plus, combine, sum
Homophones	sum/some, whole/hole
Difficult expressions	If . . .then, given that . . .
Prepositions	divided into vs. divided by, above, over, from, near, to, until, toward, beside
Comparative constructions	If Amy is taller than Peter, and Peter is taller than Scott, then Amy must be taller than Scott.
Passive structures	Five books were purchased by John.
Conditional clauses	Assuming ***X*** is true, then ***Y***

Language function words	To give instructions, to explain, to make requests

(Adapted from Steinhardt Department of Education [New York] 2009)

- Text analysis
    - Word problems often pose a challenge because they require reading and comprehension of the text, identifying the question, creating a numerical equation, and solving that equation. Reading and understanding written content in a word problem is difficult for many ELs.

When addressing the factors that affect ELs in instruction, it is essential for teachers to know their EL students and what English language development proficiency level descriptor applies to them. The Emerging, Expanding, and Bridging levels identify what a student knows and can do at a particular stage of English language development and can help teachers differentiate their instruction accordingly. These seven types of factors are only barriers for EL students if they are not addressed by teachers.

It is a common misconception that mathematics is limited to numbers and symbols. Mathematics instruction is often verbal or through text that is written in academic, not every day, language. “The skills and ideas of mathematics are conveyed to students primarily through oral and written language—language that is very precise and unambiguous” (Francis and others (1) 2006, 35). Words that have one meaning in everyday language have a different meaning in mathematics. For example, in ordinary usage the phrase “in general” often flags an exception to the norm (e.g., “The cat is shy today, but in general, she is friendly.) In contrast, “in general” is typically used in mathematics as a signpost for a universally true inductive conclusion (e.g., “…and in general, two times a whole number is an even number.) The two usages are nearly opposites. Also, many individual words, like root, point, and table, have technical meanings in mathematics that are different from what a student might use in nonacademic contexts. Reading a mathematics text can be difficult because of the special use of symbols and spatial aspects of notations, such as exponents and stacked fractions, diagrams, and charts, and the structural differences between informational and narrative text, the latter with which students are often more familiar. For example, a student might misread 5^2 (five squared) as 52 (fifty-two). Language difficulties also occur when students solve problems using algebraic language and expressions that are necessary for Mathematics I and Algebra 1 and other higher mathematics courses, such as seven less than 22; two times as large as 12; four less than four times as large as 10; one fifth of 20; ten less than the sum of 15 and 3. (Thompson 1998)

Helping all students meet the language demands of mathematics will require careful planning; attention to the language demands of each lesson, unit, and module; and ongoing monitoring of students’ understanding and ability to communicate what they know and can do. As students explore mathematical concepts, engage in discussions about mathematics topics, explain their reasoning, and justify their procedures and conclusions, the mathematics classroom will be vibrant with conversation.

Assessment to Identify Instructional Needs

One of the first tasks required of a school district is to determine its students' current achievement levels in mathematics so that each student or group of students can be offered mathematics instruction leading to the attainment of all the grade-level or course-level mathematics standards. The concept that what the student has already learned in mathematics should form the basis for further learning and study is particularly true considering the vertical alignment of the CA CCSSM. Assessments must also identify students' misconceptions and over-generalizations so that they can be corrected. (For additional information, see the "Assessment" chapter.)

FORMATIVE ASSESSMENT is key to ensuring that all students are provided with mathematics instruction designed to help them progress at an appropriate pace from what they already know to higher levels of learning. Knowing what students have learned, teachers and administrators can better plan the instructional program for each student or for groups of students with similar needs. Regardless of how students are grouped, formative assessment can be used to (1) determine which mathematical skills and conceptual understandings the student has already acquired; (2) indicate what the student needs to learn next; and (3) identify student misconceptions. With ongoing progress monitoring supported by formative assessments, student groupings remain flexible as students move in and out of groups as their instructional needs change.

DIAGNOSTIC ASSESSMENT of students often reveals both strengths and weaknesses, or gaps, in their learning. It could also reveal learning difficulties and the extent to which limited English language proficiency is interfering with learning mathematics. Once the gaps are discovered, instruction can be designed to remediate specific weaknesses while taking into consideration identified strengths. With effective support, students' weaknesses can be addressed without slowing down the students' mathematics learning progression.

For example, the development of fluency with division using the standard algorithm in grade six is an opportunity to identify and address learning gaps in place value understanding. This approach, in which instruction and learning of place value supports students' fluency with division, is more productive than postponing grade-level work to focus on earlier standards in place value (CCSSI 2010, 12). Assessments may also indicate that a student already possesses mathematical skills and conceptual understanding beyond that of his/her peers and requires a modified curriculum to remain motivated.

Successful Diagnostic Teaching

If a student is struggling unproductively to complete grade-level tasks, the teacher needs to determine the cause of the student's lack of achievement. Contributing factors might include a lack of content area knowledge, limited-English proficiency, lack of motivation, or learning difficulties. Frequent absences from school, homelessness, family issues, or reading difficulties could be factors in a student's lack of achievement. Teachers need to know their students to address their instructional needs. Often a student placed in a class or program lacks the foundational skills and conceptual understandings necessary to complete new assignments successfully. Sometimes a student may have a persistent misunderstanding of mathematics, or the student may have practiced an error consistently until it has become routine. These problems may affect his or her ability to understand and solve problems. For these struggling students, intervention may be necessary.

Task Analysis

When teaching mathematical concepts and skills, Ashlock (1998) suggests that teachers begin with what students already know, and then build upon that knowledge. For example, fluency with division is helped or hampered dependent upon students' fluency with multiplication facts. Formative assessment can help determine the multiplication facts mastered and therefore help to tailor concept instruction of division to specific multiplication facts. As students master additional multiplication facts, these multiplication facts become incorporated into the division lessons.

Additionally, Vaughn and Bos (2012) advocate that mathematics material be arranged into "smaller, more manageable amounts" so that students will be successful in learning the mathematics (Vaughn and Bos 2012, 368). These authors suggest that teachers utilize the process of task analysis to identify the prerequisite skills that students need to know before learning the mathematical concept. For example, in order for students to be able to complete a word problem with two-place addition, students will need to know:

- "Number concepts for 0-9
- Number concepts for 10-100
- Place value
- Simple oral word problems, requiring addition knowledge for 0-9

- Simple written word problems, requiring addition knowledge for 0-9
- Two-place addition problems
- Oral-addition word problems requiring knowledge of two-place addition
- Written-addition word problems requiring knowledge of two-place addition" (Vaughn and Bos 2012, 398).

Through this identification process and then determining where students are located along the skill continuum, teachers will be able to focus their teaching on those skills necessary for students to learn the targeted mathematical skill or concept.

Multi-tiered Systems of Support/Response to Instruction and Intervention (MTSS/RtI^2)

Response to Instruction and Intervention (RtI^2) is California's[2] system of good first instruction, early identification, prevention, and support for struggling students with the primary goal of preventing students from falling behind their peers. RtI^2 is a comprehensive system of effective instruction and earlier intervention for students experiencing difficulty learning to ensure that they are not misdiagnosed or over identified for special education services due to lack of appropriate instruction.

Multi-tiered systems of support (MTSS) expands the systems and processes of RtI^2 to create a comprehensive framework that leverages the principles of RtI^2 and PBIS (Positive Behavioral Interventions and Supports) as well as other interventions, supports, and services provided to assist struggling learners into a system-wide

[2] Response to Intervention (RTI) is a widely utilized system of response to students' instructional needs. In California, that systematic approach was broadened to focus on good first instruction—instruction that is carefully planned to meet the needs of all students.

continuum of resources, strategies, structures and practices to address barriers to student learning. The foundational structures of MTSS include the premise that quality core instruction utilizing UDL principles with appropriate supports, strategies, and accommodations is a basis design necessity for all instruction within Tier 1 as well as across Tiers 2 & 3. In addition, a system of assessments and progress monitoring allows for a data-based, problem-solving approach to analyzing student data to determine instructional adjustments. Providing high-quality curriculum and instruction that is sensitive to the needs of individuals are essential components within the structure of MTSS. As such, the notion of shared responsibility is particularly crucial. All students are everyone’s responsibility. Teachers must have the support of one another, administrators, specialists, parents and the community in order to best serve students. MTSS best occurs in the context of excellent curricula, effective instruction, and a comprehensive assessment system as well as effective leadership, professional learning, and an empowering culture. Schools and districts should have in place a well-defined framework for MTSS, including leadership and organizational structures, routines for program evaluation and progress monitoring of students, initial and ongoing professional learning for all educators, and clear two-way communication with parents and caregivers.

The design features of the three tiers of increasing levels of support and intensity of RtI[2] are included within the framework of MTSS. These tiers reflect the intensity of instruction, not specific programs, students, or staff (i.e., Title 1 or special education). The tiers are discussed here. The three-tiered approach is a continuum of services, both academic and behavioral, with each tier part of an interrelated process. Instructional practices are evaluated and adjusted based on results of frequent, valid, and sensitive indicators of student outcomes. While Tier 1 core high quality instruction is the foundation, each tier is critical to the overall success of the RtI^2 framework. The following sections provide descriptions of the three tiers of RtI^2 implementation.

Tier 1

Tier 1 core/universal instruction, also known as “first teaching,” is differentiated instruction delivered to all students in general education. The goal is for all students to receive high-quality standards-based instruction, with culturally and linguistically responsive curriculum, which meets the full range of student needs, from intervention to enrichment. Valid universal screenings that identify students at risk of academic and behavioral failures are reliably administered to ensure classroom-level interventions allow all students to benefit from core instruction.

Tier 2

Tier 2 is strategic/targeted instruction for students who are not progressing or responding to Tier 1 efforts as expected. At the elementary level, targeted instruction could be delivered daily for thirty minutes in small groups for six to eight weeks. At the secondary level, Tier 2 support could include a course with fewer students where on a daily basis students are pre-taught or retaught concepts taught in the core instruction. In both elementary and secondary settings, targeted students are provided with more time and more focused instruction directed to specific learning needs with more frequent monitoring of the student’s progress toward meeting identified goals. Tier 2 instructional supports are provided to students in addition to what they receive in Tier 1. The supplemental instruction provided in Tier 2 can be an extension of the core curriculum utilized in Tier 1 or may include instruction and materials specifically designed for intervention. For example, Tier 2 interventions may focus on in-depth treatment of whole number in kindergarten through grade 5 and on rational numbers in grades 4 through 8. Instruction provided during the intervention should be explicit and systematic to include providing models of proficient problem-solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review (Woodward, Beckmann, Driscoll, Frnke, Herzig, Jitendr, Koedinger, & Ogbuehi, 2012).

Tier 3

Tier 3 consists of intensive intervention instruction with continuous progress monitoring. Tier 3 interventions are for students who have difficulties with the grade-level standards in the general education curriculum and have not benefited from Tier 2 interventions and, therefore, need more intensive interventions. Tier 3 instruction should provide skill and concept development which supports and provides access to grade-level or course-level standards. It may occur in a learning center or may be at a different pace than Tier 2 instruction. The instruction for elementary students in Tier 3 may be for forty to sixty minutes daily for a period of six to eight weeks, though some students may need intensive intervention for longer periods of time. For secondary students, Tier 3 intervention is most often a double block of daily instruction for a semester or longer. In both elementary and secondary settings, the instructional goal is to provide research-based intervention more often and for longer periods of time with reduced student/teacher ratios intended to accelerate students' progress and return them to their core instructional programs (Tier 1). (Adapted from Ventura County Office of Education 2011.)

Tier I, Tier II, and Tier III Mathematics Interventions

With the caveat that there has been little research on effective RtI interventions for mathematics, Gersten, et al., provide eight recommendations (See table below) to identify and support the needs of students who are struggling in mathematics[3]. The authors note that systematic and explicit instruction is a "recurrent theme in the body of scientific research." They cite evidence for the effectiveness of combinations of systematic and explicit instruction that include teacher demonstrations and think alouds early in the lesson, unit, or module; student verbalization of how a problem was solved; scaffolded practice; and immediate corrective feedback (Gersten and others 2009). In instruction that is systematic, concepts are introduced in a logical, coherent order and students have many opportunities to apply each concept. As an example, students develop their understanding of place value in a variety of contexts before learning procedures for addition and subtraction of two-digit numbers. To help students learn to communicate their reasoning and the strategies they used to solve a problem, teachers model thinking aloud and ask students to explain their solutions. These recommendations fit within the overall framework of MTSS described above.

[3] For additional information on the eight recommendations and detailed suggestions on implementing them in the classroom, see Gersten and others 2009.

[Note: These recommendations need to be in a box or otherwise separated with graphics.]

Tier 1

Recommendation 1. Screen all students to identify those at risk for potential mathematics difficulties and provide interventions to students identified as at risk.

Tiers 2 and 3

Recommendation 2. Instructional materials for students receiving interventions should focus intensely on in-depth treatment of whole numbers in kindergarten through grade 5 and on rational numbers in grades 4 through 8. These materials should be selected by committee.

Recommendation 3. Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

Recommendation 4. Interventions should include instruction on solving word problems that is based on common underlying structures.

Recommendation 5. Intervention materials should include opportunities for students to work with visual representations of mathematical ideas and interventionists should be proficient in the use of visual representations of mathematical ideas.

Recommendation 6. Interventions at all grade levels should devote about 10 minutes in each session to building fluent retrieval of basic arithmetic facts.

Recommendation 7. Monitor the progress of students receiving supplemental instruction and other students who are at risk.

Recommendation 8. Include motivational strategies in tier 2 and tier 3 interventions. (Gersten and others 2009)

Differentiation in Depth, Pacing, Complexity, and Novelty

One important aspect of Tier I instruction is the focus on planning instruction to meet the needs of a range of students. The strategies in this section are some of the ways teachers can differentiate instruction. Research indicates that a student is most likely to learn the content when the lesson presents tasks that may be “moderately challenging.” When a student can complete an assignment independently with little effort, then learning does not occur. On the other hand, when the material is presented in a manner that is too difficult, then “frustration, not learning, is the result” (Cooper 2006, 154). Advanced learner and students with learning difficulties in mathematics often require systematically planned differentiation strategies to ensure appropriately challenging curriculum and instruction. The strategies for modifying curriculum and instruction for special education or at-risk students are similar to those used for advanced learners. This section looks at four modes of differentiation: depth, pacing, complexity, and novelty. Many of the strategies presented can benefit all students, not just for those with special needs.

Depth

Depth of understanding refers to how concepts are represented and connected by learners. The greater the number and strength of the connections, the deeper the understanding. In order to help students develop depth of understanding, teachers need to provide opportunities to build on students' current understanding and assist them in making connections between previously learned content and new content (Grotzer 1999).

Differentiation is achieved by increasing the depth to which a student explores a curricular topic. The CA CCSSM raise the level of cognitive demand through the Standards for Mathematical Practice (MP) as well as grade-level and course-level Standards for Mathematical Content. Targeted instruction is beneficial when it is coupled with adjusting the level of cognitive demand (LCD). The LCD is the degree of thinking and ownership required in the learning situation. The more complex the thinking and the more ownership (invested interest) the students have for the learning, the higher the LCD. Likewise, lower LCD requires straightforward, more simplistic thinking and less ownership by the students. Having high expectations for all students is critically important; however, posing consistently high LCD can actually set some students up for failure. Similarly, posing consistently low LCD for students is disrespectful to the students. To meet the instructional needs of the students the LCD must be adjusted at the time of instruction (Taylor-Cox 2008). One strategy that teachers can use is tiered assignments with varied levels of activities to ensure that students explore the same essential ideas at a level that builds on their prior knowledge and prompts continued growth.

Pacing

Pacing is perhaps the most commonly used strategy for differentiation. That is, the teacher slows down or speeds up instruction. This strategy can be simple, effective, and inexpensive for many students with special needs (Benbow and Stanley 1996; Geary 1994). An example of pacing for advanced learners is to collapse a year's course into six months by eliminating material the students already know (curriculum compacting) without sacrificing either depth of understanding and application of mathematics to novel situations. Or students may move on to the content standards for the next grade level (accelerating). Caution is warranted to ensure that students are not placed in above-grade level courses, in particular placing unprepared students in Mathematics I or Algebra I at middle school. (See "Appendix A: Course Placements and Sequences" for additional information and guidance.) Two recent studies on middle school mathematics course-taking report that often grade eight student are placed into Mathematics I or Algebra I courses for which they are not ready, a practice that sets up many students for failure (Finkelstein 2012 and Williams 2011).

For students whose achievement is below grade level in mathematics, an increase in instructional time may be appropriate. How much additional instructional time, both duration and frequency, depends on the unique needs of each student. Regular use of formative assessments of conceptual understanding, procedural skill and fluency, and application informs the teacher and the student about progress toward instructional goals, and instructional pacing should be modified based on the student's progress (Newman-Gonchar, Clarke, and Gersten 2009).

Complexity

Complexity is the understanding within and across the disciplines. Modifying instruction by *complexity* requires more training and skill on the part of the teacher and instructional materials that lend themselves to such variations. Complexity involves making relationships between and among ideas, connecting other concepts, and using an interdisciplinary approach to the content. When students engage in a performance task or real-world problem, they must apply their mathematical skills and knowledge and knowledge of other subjects (Kaplan, Gould, and Siegel 1995).

For students experiencing difficulty in mathematics, teachers should focus on the foundational skills, procedures, and concepts within the standards. Several studies found the use of visual representations and manipulatives can improve students' proficiency. Number lines, drawings, pictorial representations, and other types of visual representations are effective scaffolds. However, if visual representations are not sufficient, concrete manipulatives should be incorporated into instruction (Gersten and others 2009).

Teachers can differentiate the complexity of a task to maximize student learning outcomes. Differentiation for special needs students is sometimes questioned by those who say that struggling students never progress to the more interesting or complex assignments. It is important to focus on essential concepts embedded in the standards and frequent assessment to ensure that students are not just “passed along” without the understanding and skills they will need to succeed in subsequent grades. Struggling students are expected to learn the concepts well so that they develop a foundation on which further mathematical understanding can be built. Advanced students benefit from a combination of self-paced instruction and enrichment (National Mathematics Advisory Panel [NMAP] 2008).

Novelty

Keeping students engaged in learning is an ongoing instructional challenge that can be complicated by the varied instructional needs of students. Novelty is one differentiation strategy that is primarily student-initiated and can increase student engagement. Teachers can introduce novelty by encouraging students to re-examine or reinterpret their understanding of previously learned information. Students can look for ways to connect knowledge and skills across disciplines or between topics in the same discipline. Teachers can work with students to help the students learn in more personalized, individualistic, and nontraditional ways. This approach may involve a performance task or real-world problem on a subject that interests the student and requires the student to use mathematics understandings and skills in new or more in-depth ways (Kaplan, Gould, and Siegel 1995). Research by Weitzel (2008) recommends engaging struggling students in authentic experiences after mastery of skills and ensuring modeling throughout the authentic experience and include practice afterwards for students with average to above average success.

Planning Instruction for Students with Disabilities

Some students who receive their mathematics instruction in the general education classroom (Tier 1) or receive Tier 2 or Tier 3 interventions may also have disabilities that require accommodations or placements in programs other than general education. Students with disabilities tend to have difficulty remembering and retrieving basic mathematics facts. They may not be able to retain the information necessary to solve mathematics problems. Students with disabilities may continue to count on their fingers when their age-peers no longer need this kind of support. They may also have problems with writing simple equations for simple word problems and in comparing the magnitude of numbers and other basic understandings of number sense. (Gersten and others 2008)

Students with disabilities are provided with access to all the mathematics standards through a rich and supported program that uses instructional materials and strategies that best meet their needs. A student’s 504 accommodation plan or individualized education program (IEP) often includes suggestions for a variety of techniques to ensure that the student has full access to a program designed to provide him or her with mastery of the CA CCSSM, including the MP standards. Teachers must familiarize themselves with each student's 504 accommodation plan or IEP to help the student achieve mastery of the mathematics standards.

[Note: the sections on 504 plans and IEPs below will be in a separate box or otherwise separated from the other text by format.]

A Section 504 accommodation plan is typically produced by school districts in compliance with the requirements of Section 504 of the federal Rehabilitation Act of 1973. The plan specifies agreed-on services and accommodations for a student who, as a result of an evaluation, is determined to have a "physical or mental impairment [that] substantially limits one or more major life activities." Section 504 allows a wide range of information to be contained in a plan: (1) the nature of the disability; (2) the basis for determining the disability; (3) the educational impact of the disability; (4) the necessary accommodations; and (5) the least restrictive environment in which the student may be placed.

An IEP is a written, comprehensive statement of the educational needs of a child with a disability and the specially designed instruction and related services to be employed to meet those needs. An IEP is developed (and periodically reviewed and revised) by a team of individuals knowledgeable about the child's disability, including the parent(s) or guardian(s). The IEP complies with the requirements of the IDEA and covers such items as the

(1) child's present level of performance in relation to the curriculum; (2) measurable annual goals related to the child's involvement and progress in the curriculum; (3) specialized programs (or program modifications) and services to be provided; (4) participation in general education classes and activities; and (5) accommodation and modification in assessments.

Instructional Design for Students with Disabilities

In recent years, five different meta-analyses of effective mathematics instruction for students with disabilities have been conducted. The students included in these studies were most often students who have learning disabilities but also included students with mild intellectual disabilities, attention deficit/hyperactive disorder (AD/HD), behavioral disorders, and students with significant cognitive disabilities. (Adams & Carnine, 2003; Baker, Gersten, & Lee, 2002; Browder, Spooner, Ahlgrim Delzell, Harris, & Wakeman, 2008; Kroesbergen & Van Luit, 2003; Xin & Jitendra, 1999). These meta-analyses along with the National Mathematics Advisory Panel Report (2008) suggest that there are four methods of instruction that show promise for assisting students with disabilities with improving their achievement in mathematics. These instructional approaches include:

- **Systematic and explicit instruction**, where teachers guide students through a defined instructional sequence with explicit (direct) instructional practice. Often included in this direct instruction is strategy instruction where teachers model the strategy for students utilizing specific strategies including thinking aloud, mnemonics, and the problem-solving process. Teachers model the strategy so students can see when and how to use a particular strategy and what they can gain by doing so. These techniques help students learn to regularly apply strategies that effective learners use as a fundamental part of mastering concepts.

- **Self-instruction**, to guide students to learn to manage their own learning through a variety of self-regulation strategies with specific prompting or solution-oriented questions.
- **Peer tutoring**, refers to many different types of tutoring arrangements but most often involves pairing students together to learn or practice an academic task. This practice works best when students of different ability levels work together.
- **Visual representation,** which uses manipulatives, pictures, number lines, and graphs of functions and relationships to teach mathematical concepts. The Concrete-Representational-Abstract Sequence of instruction (CRA) is the most common example of the use of visual representation and one of the strategies which holds the strong promise for improving understanding of mathematical concepts for students with disabilities. CRA is an evidence-based instructional practice using manipulatives to promote conceptual understanding (Witzel, Riccomini, & Schneider, 2008). The CRA instructional sequence consists of three tiers of learning. Each tier builds upon the previous one to promote conceptual understanding and procedural accuracy and fluency.

The three tiers include: Concrete learning through hands-on instruction using actual manipulative objects; representational learning through pictorial representations of the previously used manipulative objects during concrete instruction, and lastly, learning through abstract notations such as operational symbols. Each tier interconnects with the next leading towards students becoming mathematically proficient. CRA is built upon the premise of UDL allowing for learning through multimodal forms of learning that include seeing, hearing, muscle movement, and touch. It accounts for learner variability by allowing the learner to interact in multiple ways that may in turn increase student engagement and thus a desire to attend to the task at hand. Using manipulatives in concrete and representational ways helps the learner to gain meaning from abstract mathematics by being able to break down the steps into understandable concepts. To that end the, CRA instructional sequence can help students to generalize offering a more meaningful and contextually relevant solution to rote memorization of algorithms and rules taught in isolation of the purpose of the computation.

In order to improve mathematics performance in students with learning difficulties, Vaughn, Bos, and Schumm (2011) also suggest that when new mathematics concepts are introduced or when students are having difficulty in learning the concept, teachers need to “begin with the concrete and then move to the abstract” (p. 385). Furthermore, the authors suggest that student improvement will occur when teachers provide:

- Explicit instruction that is highly sequenced and provides students with why the learning is important
- Assurance that students understand the teacher directions as well as the demands of the task by closely monitoring student work.
- The systematic utilization of learning principles such as positive reinforcement, varied practice, and student motivation
- The use of real-world examples that are understandable to students (Vaughn, Bos, and Schumm 2011, 385)

For students with significant cognitive disabilities, systematic instruction was found to be an effective instructional strategy. Studies focused on skills such as counting money and basic operations. Systematic instruction that was found to be effective included teacher modeling, repeated practice, consistent prompting and feedback. Students also learned by instruction in real-world settings, such as a store or restaurant (Browder and others 2008).

While direct instruction has been shown to be an effective strategy for teaching basic mathematical skills, the CA CCSSM emphasize conceptual understanding and connecting mathematics practice to the mathematical content. Helping students develop mathematical practices, including analyzing problems and persevering in solving them, constructing their own arguments and critiquing others, and reasoning abstractly and quantitatively, require a different approach. Based on their work with students with disabilities and students working below grade level, Stephan and Smith (2012) offer suggestions on creating a standards-based learning environment. Choosing appropriate problems, the role of the teacher(s), and the role of the students are three key components of a standards-based learning environment. The problems students are asked to solve must be carefully chosen to engage students, open-ended, and rich enough to support mathematical discourse.

Stephan and Smith recommend that problems be “grounded in real-world contexts” (Stephan and Smith 2012, 174) as well as accessible to all students and require little direct instruction to introduce. The teacher introduces the problem to be solved, reminds students of what they have already learned that may help them with the problem, and answers clarifying questions. The teacher does not provide direct instruction, but quickly sets the context for the students’ work. To foster student discussion, the teacher takes the role of information gatherer and asks questions of the students that help them reason through a problem. If students are working in small groups, the teacher moves from group to group to ensure all students are explaining their reasoning and asking their peers for information and explanations. Students take on the role of active learners who must figure out how to solve the problem instead of being told the steps to follow to solve it. They work with their peers to solve problems, they analyze their own solutions, and they apply previous learning to new situations. Depending on the problem posed, they find more than one possible answer and more than one way to solve the problem. When teachers utilize diverse parings (e.g., students working at or above grade level with students who are not) for group work, students can accomplish content- or language-task goals as well as mathematics goals. Collaborative work between the partners will facilitate inclusion through the learning of mathematical content. Vaughn, Bos, and Schumm (2011) note that collaborative learning has proven to be an effective method of instruction for students with developmental disabilities in the general education classroom.

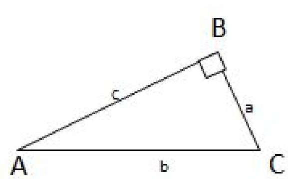
Patterns of Error in Computation

Vaughn, Bos, and Schumm (2011) indicate that many of the computation errors made by students fall into certain patterns. Ashlock (1998) theorizes that errors are generated when students “overgeneralize” during the learning process. On the other hand, other errors occur when students “overspecialize” during the learning process by restricting procedures in solving the problem (Ashlock 1998, 15). To diagnose the computational errors of students who are experiencing difficulty, assessment tools must alert the teacher to both overgeneralization and overspecialization. Teachers need to probe deeply as they examine written work—looking for misconceptions and erroneous procedures that form patterns across examples—and try to find out why specific procedures were learned. These discoveries will help teachers plan for and provide instruction to meet the needs of their students.

Errors also occur when students have not learned their basic facts, perform the incorrect operation, do not complete the algorithm in the correct sequence, lack understanding of place value within the algorithm, or provide a random response. Interviews with students regarding how they solved a problem can provide teaches with insights on students’ misunderstandings or learning difficulties. Remediation strategies that teachers can employ include returning to simpler problems, analyzing student errors and bringing to light students' misconceptions, estimating, demonstrating with concrete models to develop conceptual understanding, using grid paper so students can align numbers by place value, designing graphic organizers and flow charts, and providing students with meaningful opportunities and sequential practice to learn their basic facts for fluency.

Despite needing remediation in the elementary years, students with mathematical disabilities can successfully study higher math. The elementary mathematical curriculum focuses heavily on computation, which can place stress on students' weak memory, procedural sequencing, verbal self-monitoring, and automatization, rather than drawing on their underlying spatial strengths.

Examples of Student Error Patterns

<table>
<tr>
<td>The student thinks that $2y = 20 + y$ because $23 = 20 + 3$</td>
<td>
$c^2 = a^2 + b^2$ citing Pythagorean Theorem</td>
<td>$4 + 2 = 6$
$6 – 2 = 4$
The student does not pay attention to the addition and subtraction signs and thinks both answers are sums because they appear to the right side of equal sign.</td>
<td>Overgeneralization</td>
</tr>
<tr>
<td colspan="2">The student writes
100.36 + 12.57 as
100.36 + 125.70
because the two addends must be the same number of digits on either side of the decimal point</td>
<td>Altitude of a triangle has to be contained within the triangle</td>
<td>Overspecialization</td>
</tr>
</table>

.			
47 +34 71	65 + 36 91	37 + 25 52	The composed ten is not added. The student may be composing the ten in his/her head and forgetting to add it or he/she may be adding left to right and does not know what to do when the addition results in a two-digit answer, so he/she records only the ones digit. An interview with the student would provide further diagnostic information. (Miller 2009, 230)
45 - 37 12	46 - 28 22	36 - 17 21	The student does not decompose the tens when needed. Instead, he subtracts the smaller ones number from the larger ones number. (Miller 2009, 230)
25 x 32 50 75 125		53 x 37 371 159 530	The student misaligns the second partial product. (Miller 2009, 230)

Accommodations for Students with Disabilities

"The Standards should...be read as allowing for the widest possible range of students to participate fully from the outset and as permitting appropriate accommodations to ensure maximum participation of students with special education needs. For example, for students with disabilities *reading* should allow for the use of Braille, screen-reader technology, or other assistive devices, while *writing* should include the use of a scribe, computer, or speech-to-text technology. In a similar vein, *speaking* and *listening* should be interpreted broadly to include sign language." (CCSS ELA 2010, Introduction)

Most students who qualify for special education services will be able to achieve the standards when the following three conditions are met:

1. Standards are implemented within the foundational principles of Universal Design for Learning.
2. A variety of evidence-based instructional strategies are considered to align materials, curriculum, and production to reflect the interests, preferences, and readiness of diverse learners maximizing students' potential to accelerate learning.
3. Appropriate accommodations are provided to help students access grade-level content.

Accommodations support equitable instruction and assessment for students by lessening the effects of a student's disability. Without accommodations, students with disabilities may have difficulty accessing grade level instruction and participating fully on assessments. When possible, accommodations should be the same or similar across classroom instruction, classroom tests, and state and district assessments. However, some accommodations may be appropriate only for instructional use and may not be appropriate for use on a standardized assessment. It is crucial that educators are familiar with state policies regarding accommodations used for statewide assessment.

There are a small number of students with significant disabilities who will struggle to achieve at or near grade level. These students, who will participate in the alternative assessment, account for approximately one percent of the total student population. Substantial supports and accommodations are often necessary for these students to have meaningful access to the standards and standards-aligned assessments that are appropriate to the students' academic and functional needs. These supports and accommodations ensure that students receive access to the learning and have opportunities to demonstrate knowledge through multiple means, but retain the rigor and high expectations of the CA CCSSM.

Accommodations play an important role in helping students with disabilities access the core curriculum and demonstrate what they know and can do. The student's IEP or 504 Plan team determines the appropriate accommodations for both instruction and state and district assessments. Decisions about accommodations must be made on an individual student basis, not on the basis of category of disability or administrative convenience. For example, rather than selecting accommodations from a generic checklist, IEP and 504 Plan team members (including families and the student) need to carefully consider and evaluate the effectiveness of accommodations for each student.

Accommodations are typically made in presentation, response, setting, and timing/scheduling so that learners are provided equitable access during instruction and assessment.

- **Presentation**: Accommodations in presentation allow students to access information in ways that do not require them to visually read standard print. These alternate modes of access are auditory, multi-sensory, tactile, and manual. For example, a student with a visual impairment may require that the test be presented in a different manner, such as digital format accompanied with text-to-speech software application or the use of a Braille test booklet.
- **Response**: Accommodations in response allow students to complete activities, assignments, and assessments in different ways or to solve or organize problems using some type of assistive device or organizer. For example, a student may require an alternative method of completing multi-step computational problems due to weak fine motor skills or physical impairments, such as computer access with a specialized keyboard or speech-to-text application or specialized software to complete the task.
- **Setting**: Accommodations in setting allow for a change in the location in which a test or assignment is given or the conditions of an assessment setting. For example, a student may require that an assessment be administered in a setting appropriate to the student’s individual needs, such as testing an individual student separately from the group to accommodate accessibility such as visual and or auditory supports
- **Timing and Scheduling**: Accommodations in timing and scheduling allow for an

increase the typical length of time to complete an assessment or assignment and perhaps change the way the time allotted is organized. For example, a student may take as long as reasonably needed to complete an assessment, including taking portions over several days to avoid fatigue due to a chronic health condition.

The Council of Chief State School Officers provides guidance in its *Accommodations Manual: How to Select, Administer, and Evaluate Use of Accommodations for Instruction and Assessment of Students with Disabilities* (Thompson, Morse, Sharpe, and Hall 2005).

The selection and evaluation of accommodations for students with disabilities who are also ELs must include collaboration among educational specialists, the classroom teacher, teachers providing instruction in English language development, families, and the student. It is important to note that ELs are disproportionately represented in the population of students identified with disabilities. This suggests that some of these students may not in fact have disabilities, rather that the identification process is inappropriate for ELs.

Accommodations are available to all students including students both with and without disabilities. They do not reduce learning expectations; they provide access. Accommodations can reduce or even eliminate the effects of a student's disability. It is important to note that although some accommodations may be appropriate for instructional use, they may not be appropriate for use on a standardized assessment.

Assistive Technology

Promoting a culture of high expectations for all students is a fundamental goal of the CA CCSSM. To ensure access to the general education curriculum and CA CCSSM, students with disabilities may be provided additional supports and services, as appropriate, such as: instructional supports for learning based on the principles of UDL; instructional accommodations, modifications or changes in materials or procedures that do not change the standards but allow students to learn within the framework of the standards; and assistive technology devices and services. Assistive technology should be an important consideration within all of these areas.

Teachers implement accommodations and modifications in mathematics instruction in numerous ways, including using assistive technology. Students with physical, sensory, or cognitive disabilities may face additional challenges to learning. Students with fine motor disabilities may not be able to hold a pencil to write answers on a test or use a standard calculator to do mathematics problems. Students who have difficulty decoding text and symbols may struggle to comprehend text. When assistive technology is appropriately integrated into the classroom, students are provided with a variety of ways to access the information and to complete their work.

Disabilities vary widely, and accommodations must be tailored to the student's individual and unique needs. Assistive technology is technology used by individuals to gain access and perform functions that might otherwise be difficult or impossible. Assistive technology is defined by federal law in the *Individuals with Disabilities Education Improvement Act of 2004* as: "…any item, piece of equipment, or product system whether acquired commercially off the shelf, modified, or customized, that is used to increase, maintain, or improve functional capabilities of individuals with disabilities" (Pub. L. No. 108-466, Part A, Sec. 602, 11-12). Assistive technology can include a wide variety of learning enhancements, including mobility devices, writing implements, communication boards, and grid paper, as well as hardware, software, and peripherals that assist in accessing standards, curriculum, and instruction. For more information on assistive technology, visit http://www.washington.edu/accessit/articles?109/.

Assistive technology has several possible functions:

- Accommodation—assistive technology provides access to the course curriculum. Students can receive assistance from a computer that scans and reads text or digital content to incorporate images, sound, video clips, and additional information. Digital large print with a contrasting background, the ability to change the font as it appears on the screen, or text-to-speech devices can provide access for students with visual impairments. Software that converts text to braille characters, using a refreshable display, provides students access to printed information. Students can use mobile devices to create or record notes that they can later transfer to a computer for printing out assignments or use to study for a test. A student with motor difficulties might use an enlarged or simplified computer keyboard, a talking computer with a joy stick, switch, headgear, or eye selection devices. Augmentative and Alternative Communication (http://www.asha.org/public/speech/disorders/AAC/) systems or applications are used to help students with severe speech or language disabilities express thoughts, needs, or ideas. These and other types of assistance can provide access, but they do not change the content and are therefore considered to be accommodations.

- Modification—assistive technology provides additional scaffolding of lessons, or software that can display the main idea. Examples of modifications include the use of certain types of sign language (text-symbol) that is concept derived and dictionaries, calculators, number line, or other devices that provide information not otherwise available to students.

Although assistive technology helps to level the playing field for students with special needs, many types of assistive technology (both software and hardware) are beneficial for all students. The flexibility of assistive technology allows a teacher to use tools and materials that support students' individual strengths and also address their weaknesses in the least restrictive environment.

The California Department of Education (CDE) provides information that clarifies basic requirements for consideration and provision of assistive technology and services to each individual with a disability. Information also is available for local education agencies, particularly members of Individualized Education Program (IEP) teams, to effectively address these requirements. Visit the CDE Competencies for Assistive Technology Providers at: Assistive Technology Web page at http://www.cde.ca.gov/sp/se/sr/atstaff.asp/. (Accessed 12-30-2012)

For other examples of assistive technology, please visit the CDE Assistive Technology Checklist Web page at: http://www.cde.ca.gov/sp/se/sr/atexmpl.asp (Accessed 12-30-2012

Planning Instruction for English Learners

Ethnically and racially diverse students make up approximately 74 percent of California's student population, making our state's student population the most diverse in the nation. In 2012-13, over 1.3 million students, about a quarter of the California public school population, were identified as English learners. Of those English learners, 84.6 percent identified Spanish as their home language. The next largest group of English learners, 2.3 percent, identified Vietnamese as their home language (CDE DataQuest 2013). Given the large number of English learners in California's schools, providing effective mathematics instruction to English learners is crucial.

English learners face the double challenge of learning subject-area content at the same time they are developing proficiency in English. California law requires that instruction for most English learners be presented overwhelmingly in English. A variety of instructional settings are available to English learners, including structured English immersion (SEI), mainstream English language, and dual language instruction. The instructional methodologies for English learners vary, such as English language development (ELD), English language development and Specially Designed Academic Instruction in English (ELD SDAIE), and ELD instruction in dual language programs.

Instruction for English learners work best when it is planned according to the students' assessed levels of proficiency in English and their primary language, as well as their mathematics skills and understandings. Because of differing academic backgrounds and ages, some students may advance more quickly than others who require more support in their academic progress. Many districts use assessment tools such as the statewide assessment that assesses the progress of limited English-proficient students in acquiring the skills of listening, speaking, reading, and writing in English. The statewide assessment is designed to identify students' proficiency in English and to assist teachers in planning initial instruction, monitoring progress, and conducting summative evaluations.

The role of English language proficiency must be considered for English learners who are experiencing difficulty in learning mathematics. Even students who have good conversational English skills may lack the academic language necessary to fully access mathematics curriculum (Francis and others 2006 (1)). Academic language, as described by Saunders and Goldenberg, "entails all aspects of language from grammatical elements to vocabulary and discourse structures and conventions" (Saunders and Goldenberg 2010, 106).

To provide effective mathematics instruction to English learners,

> ”Every teacher must incorporate into his or her curriculum instructional support for oral and written language as it relates to mathematics standards and content. It is not possible to separate the content of mathematics from the language in which it is discussed and taught.” (Francis and others 2006(1), 38)

Moschkovich cautions that communicating in mathematics is more than a matter of learning vocabulary; students must also be able to participate in discussions about mathematical ideas, make generalizations, and support their claims. She states, “While vocabulary is necessary, it is not sufficient. Learning to communicate mathematically is not merely or primarily a matter of learning vocabulary” (Moschkovich 2012 (2), 18). Providing instruction that focuses on teaching for understanding, helping students use multiple representations to comprehend mathematical concepts and explain their reasoning, and supporting students’ communication about mathematics is challenging (Moschkovich 2012 (1)). Moschkovich’s recommendations for connecting mathematical content to language are provided in the table below.

Recommendations for Connecting Mathematical Content to Language
• Recommendation #1: Focus on students’ mathematical reasoning, not accuracy in using language. • Recommendation #2: Shift to a focus on mathematical discourse practices, move away from simplified views of language.

- Recommendation #3: Recognize and support students to engage with the complexity of language in math classrooms.
- Recommendation #4: Treat everyday language and experiences as resources, not as obstacles.
- Recommendation #5: Uncover the mathematics in what students say and do. (Moschkovich 2012 (1), 5-8).

To support English learners as they learn both mathematics and academic language:

•Explicitly teach and incorporate into regular practice academic vocabulary for math. Be aware of words that have multiple meanings such as root, plane, or table.

•Provide communication guides, sometimes called sentence frames, to help students express themselves not just in complete sentences but articulately within the MP standards.

•Use graphic organizers and visuals to help students understand mathematical processes and vocabulary.

Elementary school English learners' progress in mathematics may be supported through the intentional lesson planning for content, mathematical practice, and language objectives. Language objectives "…articulate for learners the academic language functions and skills that they need to master to fully participate in the lesson and meet the grade-level content standards" (Echevarria, Short, & Vogt 2008). In mathematics, students' use of the MP standards require students to translate between various representations of mathematics and to develop a command of receptive (listening, reading) and generative (speaking, writing) language. Language is crucial for schema-building; learners construct new understandings and knowledge through language, whether unpacking new learning for themselves or justifying their reasoning to a peer.

The following are examples of possible language objectives for a student in grade two:
-Read addition and subtraction expressions fluently.
-Explain the strategies and/or computational estimates used to solve addition and subtraction problems within 100.
-Describe the relationship between multiplication and division.

Francis, et al., examined research on instruction and intervention in mathematics for English learners. They conclude that there is general agreement that a lack of development of academic language is a primary cause of English learners academic difficulties and that more attention needs to be paid to its development. Like Moschkovich, Francis, et al., make clear that academic language involves many skills besides vocabulary. It includes using increasingly complex words, comprehending sentence structures and syntax, and understanding the organization of text.

One approach to helping improve students' academic language is to "amplify, rather than simplify," new vocabulary and mathematical terms (Wilson 2010). When new or challenging language is continually simplified for English learners, they cannot gain the academic language necessary to learn mathematics. New vocabulary, complex text, and the meanings of mathematical symbols are taught in context with appropriate scaffolding or amplified. Amplification helps increase students' vocabulary and makes mathematics more accessible to students with limited vocabulary. In the progression of rational number learning throughout the grades, particularly relevant to upper elementary and middle school, students encounter increasingly complex uses of mathematical language (words, symbols) that may contradict student sense-making of a term or phrase from earlier grades. For example, "half" is interpreted as either a call to divide a certain quantity by two, or to double that quantity, depending upon the context: "Half of 6 is ____?" "6 divided by one-half is___?"

The standards distinguish between number and quantity, where quantity is a numerical value of a specific unit of measure. By middle school, students are now expected to articulate that a "unit rate for Sandy's bike ride is ½ mi/hr," based upon reading the slope of a distance versus time line graph of a bike ride traveled at this constant rate. Here, "½" represents the distance traveled for each hour, rather than the equivalent ratio of one mile traveled for every two hours. The same symbols that students encountered in early elementary to represent parts of a whole (e.g., partitioning in grade 2, formalized as unit fractions in grade 3) are now attached to new language and concepts in upper elementary and middle school.

Researchers caution that focusing on academic language alone may promote teaching vocabulary without a context or lead to thinking of students as lacking because of their inability to use academic language (Edlesky 2006; MacSwan and Rolstad

2003).Instruction should move away from teaching academic language without context and instead emphasize mathematical meaning in social contexts, with an emphasis on mathematics discourse. Mathematics discourse is defined as communication that centers on making meaning of mathematical concepts; it is more than just knowing vocabulary. It involves negotiating meanings by listening and responding, describing understanding, making conjectures, presenting solutions, challenging the thinking of others, and connecting mathematical notations and representations.(Celedon-Pattichis and Ramirez 2012, 20)

Teachers' lesson planning of language, mathematical content standards, and MP standards will need to identify where these three objectives intersect and what specific scaffolds for English learners' mathematical discourse are necessary. As one example, a high school teacher of long-term English learners has planned a lesson that requires students to identify whether four points on a coordinate graph belong to a quadratic or an exponential function. Classroom routines for partner and group work have been established, and students know what "good listening" and "good speaking" look like and sound like. However, the teacher has also created bookmarks for students to use, with sentence starters and sentence frames to share their conjectures and rationales and to question the thinking of other students. After a specified time for individual thinking and writing, students share their initial reasoning with a partner. A whole class discussion ensues, with the teacher intentionally revoicing student language and asking students to share what they heard another student say in their own words. While the teacher informally assesses how students employ academic language in their oral statements, she also presses for “another way to say” or represent that thinking to amplify academic language.

The language of mathematics is not a universal language but a specialized language that requires a different interpretation than everyday language. Attention must be paid to particular terms that may be problematic. The table below provides examples of mathematical terms that may cause difficulties for English learners, depending on their context or usage.

Words with meaning only found in Mathematics (used in academic English only)	Hypotenuse, parallelogram, coefficient, quadratic, circumference, polygon, polynomial
Symbolic Language (used almost universally)	+, -, x, ÷, π, ½
Words with multiple meaning in Everyday English (EE) and Academic English (AE)	EE: The floor is even. The picture is even with the window. Sleep provides even rhythm in our breathing. The dog has an even temperament. I looked sick and felt even worse. Even a 3 year-old child knows the answer. AE: Number: Even numbers (e.g., 2, 4, 6, etc.) Number: Even amounts (e.g., even amounts of sugar and flour) Measurement: Exact amount (e.g., an even pound) Function: f (x) = f (-x)(e.g., cosine function is an even function)
Phonological words	tens vs. tenths sixty vs. sixteen sum vs. some whole vs. hole off vs. of

	How many halves do you have? then vs. than

(Table adapted from Asturias 2010)

The lack of English-language proficiency and understanding of the language of mathematics is of particular concern for long-term English learners, adolescent students who have been in American schools for many years but lack the academic language necessary to complete academic tasks and who may not be able to draw inferences, analyze, summarize, or explain their reasoning. To address the instructional needs of long-term English learners, focused instruction such as instructed English language development (ELD) may be the most effective (Dutro and Kinsella 2010). Instructed ELD, as described in Dutro, focuses attention on language learning. Language skills are taught in a prescribed scope and sequence, ELD is explicitly taught, and there are many opportunities for student practice. Lessons, units, and modules are designed to build fluency and with the goal of helping students achieve full English proficiency.

In addition to systematic ELD instruction, Dutro and Moran offer two recommendations for developing students' language in the content areas: frontloading and using teachable moments.

> Front-loading of ELD describes a focus on language preceding a content lesson. The linguistic demands of a content task are analyzed and taught in an up-front investment of time to render the content understandable to the student. This front-loading refers not only to the vocabulary, but also to the forms or

> structures of language needed to discuss the content. The content instruction, like the action of a piston, switches back and forth from focus on language, to focus on content, and back to language (Dutro and Moran 2002, 4).

One example of an instructional strategy of Dutro's "piston" that informally assess and advance students' mathematical English language development follows.

List-Group-Label

Purpose: Formative assessment of students' acquisition of academic language, and their ability to distinguish form and function of mathematical terms and symbols (e.g. the term "polygon" reminds students of types of polygons (triangles, rectangles, rhombus), or reminds students of components or attributes of polygons (angles, sides, parallel, perpendicular), or non-examples (circles)).

Process: At the conclusion of a unit of instruction, the teacher posts a mathematical category or term that students experienced in the unit and asks students to generate as many related mathematical words or symbols that they relate to the posted term as they can.

Working with a partner or group, students compile their lists of related words and agree how to best sort their list into several subgroups.

For each subgroup of terms or symbols, students must come to agreement on an appropriate label for each subgroup's list and be prepared to justify their "List-Group-Label" to another student group.

Teachers also take advantage of “teachable moments” to expand and deepen language skills. Teachers must utilize opportunities “as they present themselves to use precise language [MP. 6] to fill a specific, unanticipated need for a word or a way to express a thought or idea. Fully utilizing the teachable moment means providing the next language skill needed to carry out a task or respond to a stimulus.” (Dutro and Moran 2002, 4)

M. J Schleppegrell agrees that the language of mathematical reasoning differs from informal ordinary language. Traditionally, teachers have identified mathematics vocabulary as a challenge but are not aware of the grammatical patterning embedded in mathematical language that generates difficulties. Schleppegrell identifies these linguistic structures as “patterns of language that draw on grammatical constructions that create dense clauses linked with each other in conventionalized way” yet differ from ordinary use of language . Examples include the use of long, dense noun phrases such as *the volume of a rectangular prism with sides 8, 10, and 12 cm* , classifying adjectives that precede the noun (e.g., *prime number; rectangular prism*) and qualifiers that come after the noun (e.g. *A number which can be divided by one and itself).* Other challenging grammatical structures that may pose difficulty include conjunctions such as *if, when, therefore, given,* and *assume* which are used differently than everyday language. (Schleppegrell 2007, 143-146). Schleppegrell asserts educators need to expand their knowledge of mathematical language to include grammatical structures which enable students to participate in mathematical discourse.

Other work on mathematics discourse, such as from S. Irujo, provides concrete classroom applications for vocabulary instruction at the elementary and secondary levels. Irujo explains and suggests three steps for teaching mathematical and academic vocabulary:

- The first suggested step is for educators to read texts, tests, and materials analytically to identify potential difficulties by focusing on challenging language.
- Irujo's second step follows Dutro's findings in pre-teaching experiential activities in mathematics. Only the necessary vocabulary and key concepts are taught to introduce the central ideas at this time.
- The third and final step is integration of the learning process. New vocabulary is pointed out as it is encountered in context, its use is modeled frequently by the teacher, the cycle of modeling is repeated, followed by guided practice, small group practice, and independent practice. She recommends teaching complex language forms through mini-lessons (Anstrom and others 2010, 23).

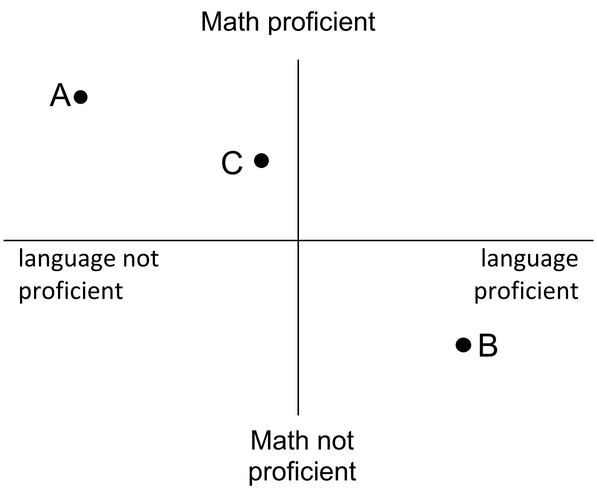
Despite the importance of academic language for success in mathematics, "…in mathematics classrooms and curricula the language demands are likely to go unnoticed and unattended to" (Francis and others 2006 (1), 37). Both oral and written language need to be integrated into mathematics instruction. All students, not just English learners, must be provided many opportunities to talk about mathematics and explain their reasoning and understanding—to engage in mathematics discourse. The language demands of mathematics instruction must be noted and attended to. Mathematics instruction that includes reading, writing, and speaking enhances students' learning. As lessons, units, and modules are planned, both language objectives and content objectives should be identified. By focusing on and modifying instruction to address English learners' academic language development, teachers support their students' mathematics learning.

Van de Walle (2007) suggests specific strategies that teachers can do to support English Learners with their mathematics instruction:

- Let students know the purpose of the lesson and what they will be accomplishing during the lesson
- Build background knowledge and link the lesson to what students already know.
- Encourage the use of native language during group work while continuing to progress in English language development.
- Cooperative groups provide English learners with opportunities to utilize language in nonthreatening ways.

COURSE PLACEMENT OF ENGLISH LEARNERS

Careful attention to placement and assessment practices is particularly important for students who have studied mathematics in other countries and may be proficient in performing higher level mathematics but lack proficiency with the English language. A student's performance on mathematics assessment will be affected by the student's language proficiency. For example in the graph below Students A, B, and C's results on the same test may look very similar, even though their language and mathematical proficiency levels vary considerably. The design of the assessment needs to be mindful of this problem, and the results need to be interpreted with the language proficiency factored in. Assessing mathematics in the student's primary language should be considered so that lack of English language proficiency does not affect the test results.



(Graphic adapted from Asturias 2010.)

For English learners who may know the mathematical content but have difficulty on assessments due to lack of proficiency with the English language, Burden and Byrd (2010) list the following strategies for adapting assessments for English learners:

- *Range*. Decrease the number of assessment items.
- *Time.* Provide extra time for English learners to complete the task.
- *Level of Support*. Increase the amount of scaffolding that is provided during the assessment.
- *Difficulty*. Adapt the problem, the task, or the approach to the problem.
- *Product*. Adapt the type of response to decrease reliance on academic language.

- *Participation*. Allow for cooperative group work and group self-assessment using student-created rubrics for performance tasks.

Celedón-Pattichis (Celedón-Pattichis 2004, 188) advises that the initial placement of English learners is highly important because “these placements tend to follow students for the rest of their academic lives.” When placement of highly proficient students is not based upon their mathematical competence but rather on their language proficiency, they may (1) lose academic learning time and the opportunity to continue with their study of higher-level mathematics and (2) experience a decline in their level of mathematics because of little practice. On the other hand, when low performing students are placed in coursework that is too difficult for their knowledge or language proficiency level, they are likely to become discouraged.

Similarly, students who have studied mathematics in other countries may “confront noticeable differences” in how mathematical concepts are represented when they enter California classrooms. Notational differences include how students read and write numbers, use a decimal point, and separate digits in large numbers. There may be differences in the designation of billions and trillions. For example,

> A student schooled in the United States will read 10, 782,621,751 as ‘10 billion, 782 million, 621 thousand, 751.’ In some students’ countries of origin, the number is read as 10 mil 782 millions, 621 mil, 751’; or it is read as ’10 thousand 782 million, 621 thousand, 751’” (Perkins & Flores, 2002, p. 347).

Algorithmic differences occur in how students compute problems by algorithm. For example, they may mentally compute the steps in an algorithm and only write the answer or display the intermediate steps differently, as with long division. Additional difficulties occur as students confront United States currency (Perkins and Flores 2002).

These differences may become apparent when parents educated in other countries assist their children at home. There is a strong need for a meaningful dialogue between parents and teachers in which learning about the different methods and approaches can occur for all. For example, when students or parents possess different ways of doing arithmetic operations, teachers can use these different approaches as learning opportunities instead of dismissing them. This is particularly important for immigrant children (or children of immigrant parents), who are often navigating two worlds. As Cummins (2000) states, "Conceptual knowledge developed in one language helps to make input in the other language comprehensible" (Civil and Menendez 2010).

Planning Instruction for Standard English Learners (SELs)

Standard English learners are students who speak a nonstandard form of English, a form of English that differs in structure and form from Standard and academic English or may be influenced by another language. The Academic English Mastery Program (AEMP) and the Multilingual and Multicultural Department of Los Angeles Unified School District (LAUSD) have identified six access strategies to help SELs to be successful:

1. Making Cultural Connections – Culturally responsive pedagogy uses the “cultural knowledge, prior experience, frames of reference and performance styles” of students to make learning more relevant, effective an engaging. (English Learner Master Plan, LAUSD, p. 85)
2. Contrastive Analysis – Comparing and contrasting the linguistic features of the primary language and Standard English. (English Learner Master Plan, p. 162). During a content lesson, the teacher may demonstrate the difference in languages by the teacher repeating the student response in Standard English. This recasting then may be used at a later date as an exemplar to examine the differences.

    In this example, note the differences in subject/verb agreement, plurals and past tense:

    Non-Standard English—There was three runner. The winner finish the race in three minute.

    Standard English: There were three runners. The winner finished the race in three minutes.

3. Cooperative Learning – Working in pairs or small groups
4. Instructional Conversations – Academic conversations, often student led, allowing students to use the language to analyze, reflect, and think critically. These conversations may also be referred to as accountable talk or handing off.
5. Academic Language Development – Explicit teaching of vocabulary and language patterns needed to express the students’ thinking. Like English learners, SELs benefit from the use of sentence frames (communication guides);

unlike the supports for ELs, the guides are based on Standard English and academic vocabulary and not on English language proficiency levels.

6. Advanced Graphic Organizers- Visual representation to help students organize thoughts.

Planning Instruction for At-Risk Learners

Mathematical focus and in-depth coverage of the CA CCSSM are as necessary for students with mathematics difficulties as they are for more proficient students (Gersten and others 2009). When students begin to fall behind in their mastery of mathematics standards, immediate intervention is warranted. Interventions must combine practice in material not yet mastered with instruction in new skill areas. Students who are behind will find it a challenge to catch up with their peers and stay current with them as new topics are introduced. The need for remediation cannot be allowed to exclude these students from instruction in new concepts. In a standards-based environment, students who are struggling unproductively to learn or master mathematics need the richest and most organized type of instruction. For some students, Tier 3 interventions may be necessary.

Students who have fallen behind, or who are in danger of doing so, may need more than the normal schedule of daily mathematics. Systems must be devised to provide these students with ongoing tutorials. It is important to offer special tutorials before or after school or on Saturday; however, to ensure access for all students, extra help and practice should occur in extra periods of mathematics instruction during the school day.

Instructional time might be extended in summer school with extra support focused on strengthening and rebuilding foundational concepts and skills that are lacking from earlier grades.

Requiring a student with intensive learning challenges to remain in a course for which he or she lacks the foundational skills to master the major concepts, and thereby to pass the course, wastes student learning time. Course and semester structures and schedules for classes should be reexamined and new structures devised, such as a two-year Mathematics I or Algebra I course, so that students enrolled in such essential courses can successfully complete the full course. Targeted intervention at the middle school level and earlier can increase students' chances of being successful in higher mathematics. Early intervention in mathematics is both powerful and effective (Newman-Gonchar, Clarke, and Gersten 2009).

Grouping as an Aid to Instruction

The first focus of educators should always be the quality of instruction; grouping is a secondary concern. Grouping is a tool and an aid to instruction, not an end in itself. As a tool, grouping should be used flexibly to ensure that all students achieve the standards, and instructional objectives should always be based on the CA CCSSM. Small group instruction may be utilized as a "temporary measure" for students who have failed to grasp prerequisite content (Emmer and Evertson, 2009). For example, a teacher may discover that some students are having trouble understanding and using the Pythagorean Theorem. Without this understanding they will have serious difficulties in higher-level mathematics. It is perfectly appropriate, even advisable, to group those students who do not understand a concept or skill, such as the Pythagorean Theorem, find time to reteach the concept or skill in a different way, and provide additional practice. At the same time those students might be participating with a more heterogeneous mix of students in other classroom activities and groups in which a variety of mathematics problems are discussed.

Teachers must rely on their experiences and judgment to determine when and how to incorporate grouping strategies into the classroom. To promote maximum learning when grouping students, educators must ensure that assessment is frequent, that high-quality instruction is always provided for all students, and that the students are frequently moved into appropriate instructional groups according to their needs.

Planning Instruction for Advanced Learners

Advanced learners, for purposes of this framework, are students who demonstrate or are capable of demonstrating performance in mathematics at a level significantly above the performance of their age group. They may include (1) students formally identified by a school district as gifted and talented pursuant to California *Education Code* Section 52200 and (2) other students who have not been formally identified as gifted and talented but who demonstrate the capacity for advanced performance in mathematics. In California it is up to each school district to set its own criteria for identifying gifted and talented students. The percentage of students so identified varies, and each district may choose whether to identify students as gifted on the basis of their ability in mathematics.

The criteria should take into account students still struggling with language barriers. The criteria should also include alternative measures to identify students who are highly proficient in mathematics or have the capacity to become highly proficient in mathematics but may have a learning disability.

When the National Mathematic Advisory Panel (NMAP) looked at research on effective mathematics instruction for gifted students, they found only a few studies that met their criteria for evaluating research. This lack of rigorous research limited the Panel's findings and recommendations, and the Panel called for more high-quality research to study the effectiveness of instructional programs and strategies for gifted students. Based on the research available, the Panel reported the following findings:

[Note: These recommendations need to be in a box or otherwise separated with graphics.]

- The studies reviewed provided some support for the value of differentiating the mathematics curriculum for students with sufficient motivation, especially when acceleration is a component (i.e., pace and level of instruction are adjusted).

- A small number of studies indicated that individualized instruction, in which pace of learning is increased and often managed via computer instruction, produces gains in learning.

- Gifted students who are accelerated by other means not only gained time and reached educational milestones earlier (e.g., college entrance) but also appear to achieve at levels at least comparable to those of their equally able same-age peers on a variety of indicators even though they were younger when demonstrating their performance on the various achievement benchmarks.

- Gifted students appeared to become more strongly engaged in science, technology, engineering, or mathematical areas of study. There is no evidence in the research literature that gaps and holes in knowledge have occurred as a result of student acceleration. (NMAP 2008).

Based on these findings and the general agreement in the field of gifted education, the Panel stated, “combined acceleration and enrichment should be the intervention of choice” for mathematically gifted students (NMAP 2008, 53). The Panel recommended that mathematically gifted students be allowed to learn mathematics at an accelerated pace and encouraged schools to develop policies that support challenging work in mathematics for gifted students. (See “Appendix A: Course Placement and Sequences” for additional guidance.)

Standards-based education offers opportunities for students who have the motivation, interest, or ability (or all of these) in mathematics to excel. Several research studies have demonstrated the importance of setting high standards for all students, including advanced learners. The CA CCSSM provide students with goals worth reaching and identify the point at which skills and knowledge should be mastered. The natural corollary is that when standards are mastered, advanced students should either move on to standards at higher grade levels, focus on unlearned material not covered by the standards, or delve deeper into mathematical concepts and connections across domains. The latter approach provides students with enrichment and depth in studying the standards for their grade level. Enrichment or extension leads the student to complex, technically sound applications. Activities and challenging problems should be designed to contribute to deeper learning or new insight.

Accelerating the learning of advanced students requires the same careful, consistent, and continual assessment of their progress as is needed to support the learning of average and struggling students. Responding to the results of such assessments allows districts and schools to adopt innovative approaches to teaching and learning to best meet the instructional needs of their students.

Care must be taken in the design of standards-based programs to avoid the errors of the past. In a common core standards-based classroom, the design of instruction demands dynamic, carefully constructed, mathematically sound lessons, units, and modules devised by groups of teachers pooling their expertise in helping children to learn. These teams must devise innovative methods for using regular assessments of student progress in conceptual understanding, procedural skill and fluency, and application to ensure each student’s progress toward mastery of the mathematics standards.

Resources

Educators may visit the following Web sites to obtain resources for understanding and addressing the needs of students with disabilities:

- Laws and Regulations: California Special Education and Related Laws (New 15-Jun-2012), Searchable database for *Education Code*, Part 30, Other Related Laws and *California Code of Regulations*, Title 5. Users may search this database to find pertinent legislation for various special education topics.

- California Department of Education Special Education Web page: Information and resources to serve the unique needs of persons with disabilities so that each person will meet or exceed high standards of achievement in academic and nonacademic skills. http://www.cde.ca.gov/sp/se/ (Accessed 12-30-2012)

- California Department of Education Competencies for Assistive Technology Providers at: Assistive Technology Web page at http://www.cde.ca.gov/sp/se/sr/atstaff.asp/. (Accessed 12-30-2012)

- California Department of Education Assistive Technology Checklist Web page at: http://www.cde.ca.gov/sp/se/sr/atexmpl.asp (Accessed 12-30-2012)

For examples of research, ideas, and assistive technology to support mathematics students, visit the Technology Matrix at: http://techmatrix.org/. Accessed 12-30-2012.

For research, examples, and resources for Universal Design for Learning, go to the National Center on Universal Design for Learning Web page at http://www.udlcenter.org/aboutudl/udlguidelines. Accessed 12-30-2012.

For resources to support English learners, go to:

The California English Language Development Standards, adopted by the State Board of Education in November 2012 (http://www.cde.ca.gov/sp/el/er/eldstandards.asp)

August 2013 Review Draft

This document is recommended by the Instructional Quality Commission for adoption by the California State Board of Education (SBE). Action by the SBE is anticipated at its November 6–7, 2013 meeting.